Analysis of RF-Pilot-based Phase Noise Compensation for Coherent Optical OFDM Systems

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Abstract—In coherent optical long-haul transmission systems orthogonal frequency-division multiplexing represents a promising modulation format. However, due to long symbol length, laser phase noise can be a major impairment. In this manuscript, the RF-pilot-based phase noise compensation scheme is analyzed and compared to conventional common-phase error compensation. It has been shown, that the RF-pilot-based phase noise compensation scheme allows for a considerable increase in tolerable laser linewidth as compared to conventional common-phase error compensation at the cost of an increase in system complexity. For a 112 Gbit/s transmission scheme, the tolerable linewidth is increased by a factor of ten as compared to common-phase error compensation.

Index Terms—Optical fiber communication, Digital modulation, Orthogonal frequency-division multiplexing, Phase noise

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a promising modulation scheme for next generation optical long-haul transmission systems. These systems need to transmit 112 Gb/s Ethernet traffic within a 50 GHz wavelength grid over distances of up to 2000 km. In recent years, a variety of coherent optical transmission schemes combined with digital signal processing (DSP) have been proposed and demonstrated, that provide a large tolerance towards impairments like chromatic dispersion (CD), polarization-mode dispersion, and cascaded filtering [1]–[3]. It is known, that in coherent optical transmission systems, laser phase noise represents a major performance impairment that must be compensated for [4], [5]. In particular because of its long symbol size, OFDM requires an effective compensation mechanism.

In the field of wireless communications the phase noise tolerance of OFDM has already been studied in detail [6]–[9]. It has been found, that its influence is twofold, i.e., it generates a common phase error (CPE) of all the subcarriers in one symbol and a cross-leakage between the subcarriers referred to as inter-carrier interference (ICI). The former effect is commonly solved by estimating the mean phase rotation of each symbol from dedicated pilot subcarriers and rotating the received symbols back. This method has also been used in several fiber-optic transmission experiments and simulations, see e.g. [2], [10]. The main drawback of CPE, however, is that it does not correct for the ICI, since it inherently assumes the phase of the transmit and the receive laser to be constant during one OFDM symbol. Consequently, the OFDM symbol must be short and the laser linewidths must be small to limit the impact of ICI. However, shorter OFDM symbols require larger cyclic-prefix overheads in order to compensate for CD and small linewidths require costly lasers. In [3] a new phase noise compensation scheme has been introduced and referred to as RF-pilot-based phase-noise compensation, which effectively compensates for both the CPE and the ICI. With this technique, phase-noise compensation is realized by placing a RF-pilot (RFP) tone in the center of the OFDM transmit spectrum, which is subsequently used at the receiver in order to undo phase noise impairments.

In this manuscript a detailed comparison between CPE-based and RFP-based phase noise compensation is presented. After introducing the underlying system model, it is shown by means of numerical simulation that the RFP-based phase noise compensation scheme allows for a considerable improvement in phase-noise tolerance and that it can be regarded as an enabling technique for 112 Gb/s PDM-OFDM transmission systems.

II. SYSTEM MODEL

In a DSP unit, the $l$th symbol of the discrete-time OFDM signal $x_{l,k}$ with variance $\sigma^2$ is generated using the $N$-point inverse discrete Fourier transform (DFT) according to

$$x_{l,k} = \frac{1}{\sqrt{N}} \sum_{n=-N/2}^{N/2} c_{l,n} \exp \left[ j \frac{2\pi n (k-\nu)}{N} \right],$$

for $k = 1, 2, \ldots, N + \nu$. The $c_{l,n}$ are complex coefficients representing either information or control symbols modulated onto the $n$th subcarrier. The dc component $c_{l,0}$, the Nyquist component $c_{l,N/2}$ and $N_x$ subcarriers with indices $|n| \geq (N - N_x)/2$ are padded with zeros, in order to relax requirements for RF components. Furthermore, a cyclic prefix (CP) of $\nu$ samples is inserted in order to avoid ISI, e.g., due to CD.

In the transmitter, two uncorrelated OFDM signals are first converted to optical signals by digital-to-analog converters with sampling rate $f_s$ and electro-optical in-phase/quadrature-modulators. Then these signals are combined using polarization-division multiplexing (PDM) and transmitted over the optical link. Finally, the signal is detected with a digital coherent receiver and fed to the receiver DSP unit, where the two polarizations are separated with the help of multiple-input multiple-output processing [11]. In the presence of laser phase-noise and amplified spontaneous emission
(ASE) noise, the received discrete-time OFDM signal for each polarization becomes

\[ \hat{x}_{l,k} = x_{l,k} \exp(j\phi_{l,k}) + n_{l,k}. \]  

The random phase shifts, \( \phi_{l,k} \), due to laser phase noise can be modeled as a Wiener process according to

\[ \phi_{l,k} = \phi_{l-1,N} + \sum_{m=1}^{k} \varphi_{l,m}, \]  

where \( \varphi_{l,m} \) is the \( m \)-th element of the \( l \)-th realization of a set of \( N + \nu \) independent real-valued Gaussian distributed random variables with zero mean and variance \( \sigma_{\varphi}^2 = 2\pi \Delta f / f_s \) [4]. In the linear channel, the parameter \( \Delta f \) represents the sum of the laser linewidths of the transmit and the receive laser. ASE noise is included by the additive zero mean complex Gaussian random variable \( n_{l,k} \) with variance \( \sigma_n^2 = \sigma_{\varphi}^2 \times f_{\text{ref}} / f_s / \text{OSNR}/2 \), where the optical signal-to-noise ratio (OSNR) is referred to a bandwidth \( f_{\text{ref}} \) corresponding to 0.1 nm area around a center frequency of 193.1 THz.

In case of CPE-based phase noise estimation, a set of \( N_p \) equally spaced pilot symbols with indices \( \{n_p\} \) is transmitted within the \( N \) subcarriers. At the receiver side, the constellations \( \hat{c}_{l,n} \) with phase noise impairment are first received with help of the DFT according to

\[ \hat{c}_{l,n} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \hat{x}_{l,k} \exp(-j \frac{2\pi kn}{N}). \]  

Afterwards, the CPE \( \phi_{c,l} \) is estimated as the mean phase rotation of the \( l \)-th received OFDM symbol by

\[ \exp(j\phi_{c,l}) = \frac{1}{N_p \ \ n_p \in \{n_p\}} \sum_{n_p} |c_{l,n}|c_{l,n}^{*} \]  

Finally, the received constellations are obtained from \( \hat{c}_{l,n} = \hat{c}_{l,n}\exp(-j\phi_{c,l}) \).

In case of RFP-based phase noise estimation, a gap of \( N_p \) zeros is inserted around dc such that \( c_{l,n} = 0 \) for all \( |n| \leq N_p/2 \) and a single pilot is inserted at dc\(^1\) such that \( |c_{l,0}|^2 = N_p/(N_m - N_p) \). The received signal is filtered with a low-pass filter defined by its \( M \)-point discrete impulse response \( h_m \) resulting in

\[ \tilde{x}_{l,k} = \frac{1}{M} \sum_{m=0}^{M-1} \hat{x}_{l,k+m}h_m. \]  

Then the phase noise is estimated as

\[ \exp(j\phi_{l,k}) = \frac{\tilde{x}_{l,k}}{\vert \tilde{x}_{l,k} \vert} \]  

and finally the received constellations are obtained from

\[ \hat{c}_{l,n} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \tilde{x}_{l,k} \exp(-j\phi_{l,k}) \exp(-j \frac{2\pi kn}{N}). \]  

Clearly, The RFB based phase noise compensation scheme does not only compensate for the CPE but also for the fast
c\(^1\)In practical systems this pilot is often shifted to RF frequencies, explaining the name given to this scheme.

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**III. PERFORMANCE COMPARISON**

In this section the performance of both phase noise compensation schemes is compared using the example of a 112 Gb/s PDM-OFDM using 8-ary quadrature amplitude modulation (QAM). In order to be able to compensate for the accumulated CD of a 2000km link with 17ps/nm/km fully in the electrical domain, a DFT-size of \( N = 2048 \) and \( \nu = 140 \) is chosen corresponding to a CP overhead of 6.84%. Notice, that the DFT size cannot be reduced without an accordant increase of the CP overhead. An additional overhead of 4% is applied for coefficient training and synchronization. For phase noise compensation \( N_p = 40 \) subcarriers are reserved in case of both compensation schemes and zero-padding is set to \( N_z = 236 \). The required sampling rate results in \( f_s = 24 \) GHz and the nominal data rate including training and cyclic prefix overheads is 124.4 Gb/s.

Fig. 1(a) depicts the electrical transmit and receive spectra with phase noise impairment of \( \Delta f = 200 \) kHz in case of the CPE-based scheme as well the location of the equally spaced pilot sub-carriers. In Fig. 1(b) the same curves are shown for RFP-based phase noise compensation with a single strong pilot at the center frequency and the signal after low-pass filtering is illustrated alongside. Fig. 1(c) shows a zoomed version of Fig. 1(b) providing a better resolution of the region of interest. The phase noise impairment can clearly be observed as a Lorentzian-shaped line-width broadening. In this example, a
Gaussian-shaped low-pass filter with a bandwidth of 32.5 MHz is used. It can clearly be seen, that the filtered pilot includes dominant parts of the phase noise spectrum, which are then applied for its compensation according to (8).

Fig. 2 shows the required OSNR for a target bit-error rate of $10^{-3}$ obtained by Monte Carlo simulation as a function of $\Delta f$ for the two phase noise compensation schemes. The simulations are based on the equations introduced in section II. In case of the CPE-based scheme, a penalty of 0.5 dB occurs at about $\Delta f = 40$ kHz while for RFP-based phase noise compensation $\Delta f = 400$ kHz can be tolerated with the same penalty. The corresponding linewidth-to-bit-rate ratios are thus $\Delta f/B = 3.28 \times 10^{-7}$ and $\Delta f/B = 3.28 \times 10^{-6}$, respectively. In [4] the linewidth tolerance of single-carrier 8-ary QAM has been found to be $\Delta f/B = 5.23 \times 10^{-5}$. Even though the symbol length for PDM-OFDM is more than 2000 times larger, the phase noise sensitivity is only increased by approximately a factor of ten. The optimum bandwidth represents a trade-off between the phase-noise dynamics, ASE noise, and interference from the subcarriers modulated with data and thus grows with increasing linewidths. In the above simulations, bandwidth was varied between 0 Hz and 50 MHz and the values corresponding to the lowest OSNR requirement are displayed in Fig. 2 as triangles.

The principle of RFP-based phase noise compensation is further illustrated in Fig. 3. An example of the temporal evolution of random phase noise with $\Delta f = 200$ kHz is shown for three consecutive OFDM symbols. Furthermore, the phase noise estimates obtained with the CPE-based as well as the RFP-based scheme are shown. Clearly, the CPE compensation accounts for the mean phase noise per OFDM symbol but does not correct for the fast fluctuations. However, these residual phase dynamics can result in considerable ICI as illustrated by the constellation diagram shown in Fig. 3. In contrast, the RFP-based scheme provides an exact estimate also of the fast phase dynamics and ICI can almost completely be compensated.

**IV. Conclusions**

It has been shown, that the RFP-based phase noise compensation scheme allows for a considerable increase in tolerable laser linewidth as compared to conventional CPE compensation at the cost of an increase in system complexity. Applied to a 112 Gb/s PDM-OFDM transmission system, simulation results reveal that the tolerable laser linewidth can be increased from 40 kHz to 400 kHz at penalty of only 0.5 dB. The RFP-based scheme thus considerably relaxes the laser phase noise requirement and can be an enabling technique for future PDM-OFDM long-haul transmission systems.

**REFERENCES**